MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate number

Thursday 6 May 2004 (afternoon)

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The polynomial $x^2 - 4x + 3$ is a factor of $x^3 + (a-4)x^2 + (3-4a)x + 3$.
Calculate the value of the constant a.

Working:

Answer:

....

2. Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x - 2x$ and $y = 3$ when $x = 0$, find an expression for y in terms of x.

Working:

Answer:

....

3. For $-3 \le x \le 3$, find the coordinates of the points of intersection of the curves

$$y = x\sin x \text{ and } x + 3y = 1.$$

Working:

Answer:

4. The three terms a, 1, b are in arithmetic progression. The three terms 1, a, b are in geometric progression. Find the value of a and of b given that $a \neq b$.

Working:

Answer:

5. The linear transformations $\boldsymbol{M}$ and $\boldsymbol{S}$ are represented by the matrices

$$\boldsymbol{M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \boldsymbol{S} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Give a full geometric description of the single transformation represented by the matrix $\boldsymbol{SMS}$.

Working:

Answer:

6. Let the complex number z be given by

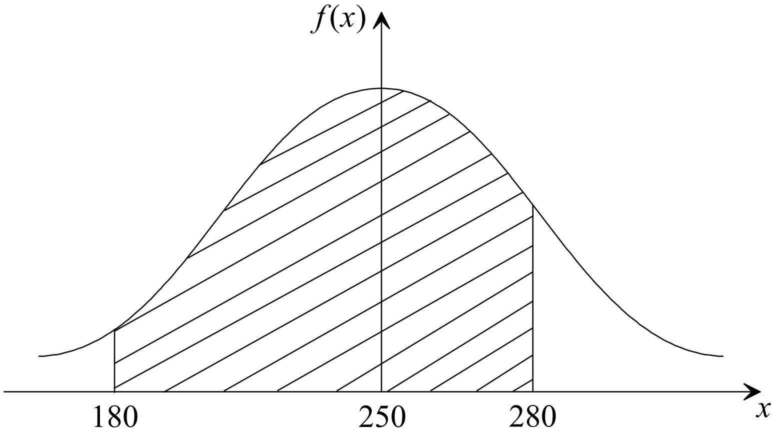
$$z = 1 + \frac{\mathrm{i}}{\mathrm{i} - \sqrt{3}}\,.$$

Express z in the form $a + b\mathrm{i}$, giving the **exact** values of the real constants a, b.

Working:

Answer:

7. The following diagram shows the probability density function for the random variable X, which is normally distributed with mean 250 and standard deviation 50.



Find the probability represented by the shaded region.

Working:

Answer:

8. The point $\mathrm{P}(1,\ p)$, where $p > 0$, lies on the curve $2x^2y + 3y^2 = 16$.

(a) Calculate the value of p.

(b) Calculate the gradient of the tangent to the curve at P.

Working:

Answers:

(a) ______________________

(b) ______________________

9. The line $\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{k} + \mu(\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$ and the plane $2x - y + z + 2 = 0$ intersect at the point P. Find the coordinates of P.

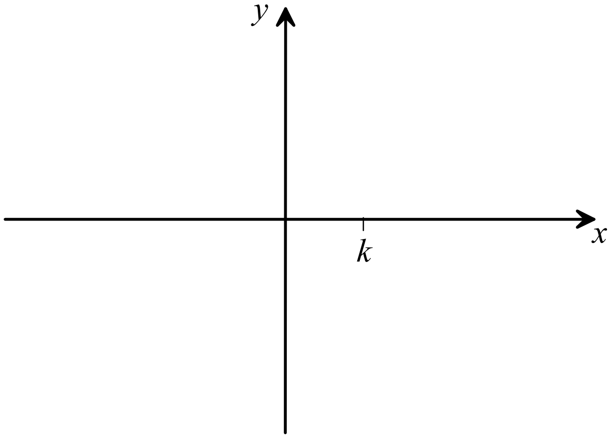
Working:

Answer:

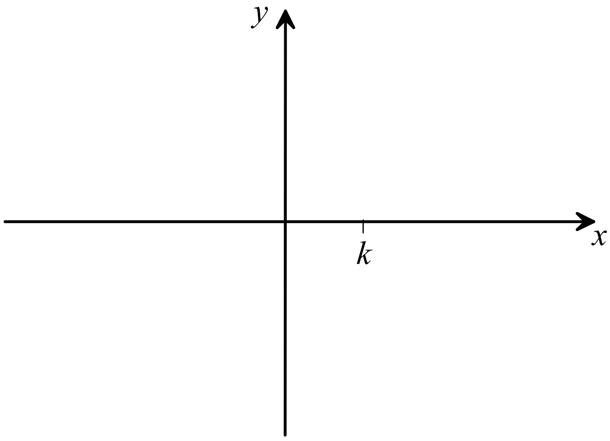
____

10. Let $f(x) = \dfrac{k}{x-k}$, $x \neq k$, $k > 0$.

(a) On the diagram below, sketch the graph of f. Label clearly any points of intersection with the axes, and any asymptotes.



(b) On the diagram below, sketch the graph of $\dfrac{1}{f}$. Label clearly any points of intersection with the axes.



Working:

11. The function f is defined by $f : x \mapsto x^3$.

Find an expression for $g(x)$ in terms of x in each of the following cases

(a) $(f \circ g)(x) = x + 1$;

(b) $(g \circ f)(x) = x + 1$.

Working:

Answers:

(a) ____________________

(b) ____________________

12. (a) Find $\int_0^m \frac{\mathrm{d}x}{2x+3}$, giving your answer in terms of m.

(b) Given that $\int_0^m \frac{\mathrm{d}x}{2x+3} = 1$, calculate the value of m.

Working:

Answers:

(a) ______________________________

(b) ______________________________

13. The discrete random variable X has the following probability distribution.

$$\mathrm{P}(X = x) = \begin{cases} \dfrac{k}{x}, & x = 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

Calculate

(a) the value of the constant k;

(b) $\mathrm{E}(X)$.

Working:

Answers:

(a) ______________________

(b) ______________________

14. Robert travels to work by train every weekday from Monday to Friday. The probability that he catches the 08.00 train on Monday is 0.66. The probability that he catches the 08.00 train on any other weekday is 0.75. A weekday is chosen at random.

(a) Find the probability that he catches the train on that day.

(b) Given that he catches the 08.00 train on that day, find the probability that the chosen day is Monday.

Working:

Answers:

(a) ______________________

(b) ______________________

15. Given that $\boldsymbol{a} = (\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) \times (-2\boldsymbol{i} + 3\boldsymbol{k})$,

(a) find $\boldsymbol{a}$;

(b) find the vector projection of $\boldsymbol{a}$ onto the vector $-2\boldsymbol{j} + \boldsymbol{k}$.

Working:

Answers:

(a) ______________________

(b) ______________________

16. Solve the inequality

$$\left|\frac{x+9}{x-9}\right| \le 2 .$$

Working:

Answer:

17. The function f is defined by $f : x \mapsto 3^x$.

Find the solution of the equation $f''(x) = 2$.

Working:

Answer:

....

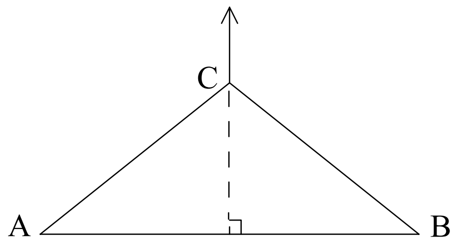
18. Find $\int \frac{\ln x}{\sqrt{x}} \mathrm{d}x$.

Working:

Answer:

....

19. The following diagram shows an isosceles triangle ABC with AB = 10 cm and AC = BC. The vertex C is moving in a direction perpendicular to (AB) with speed 2 cm per second.



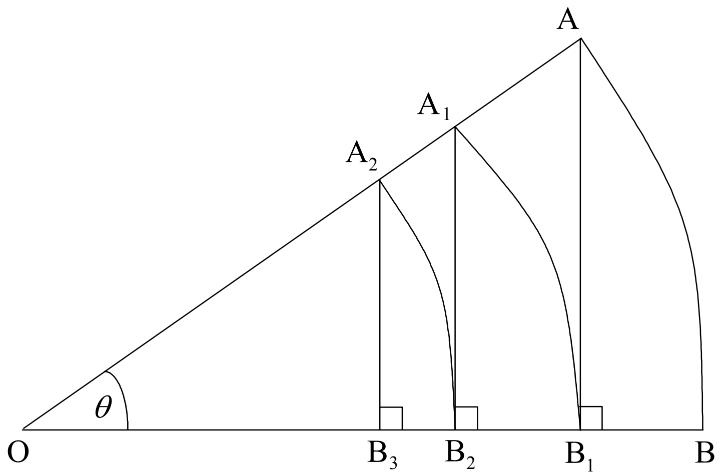
Calculate the rate of increase of the angle CAB at the moment the triangle is equilateral.

Working:

Answer:

20. The diagram shows a sector AOB of a circle of radius 1 and centre O, where $\hat{AOB} = \theta$.

The lines $(AB_1), (A_1B_2), (A_2B_3)$ are perpendicular to OB. A_1B_1, A_2B_2 are all arcs of circles with centre O.



Calculate the sum to infinity of the arc lengths

$$AB + A_1B_1 + A_2B_2 + A_3B_3 + \ldots$$

Working:

Answer: